Empty Spacetime Contains Form of Metric

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Abstract

Euclidean geometry, inherited from ancient Greece, was modeled on axiomatic methods in modern science. Hilbelt's "Foundations of Geometry" supplemented the lacking axioms, and seemed to have reached the stage of completion as plane geometry, but still questioned why the axiom system did not depart from the properties of the plane itself. Looking back on the history of special relativity, Lorentz and Poincaré were on their way to give a mathematical proof to the results of the Michelson-Morley experiment. Meanwhile Einstein published the special theory of relativity based on the invariant speed of light. At a glance, all things had been well done by this theory, but it is not enough. Digging into why the principle holds, we arrived at a deeper symmetry of spacetime. Considering the ways of Heaven, we observed that the fundamental symmetry in the cosmos is a plane with back and front symmetric surface in which the basic laws of nature must be subject to this symmetry.

Outline of the Mathematical Process[1]

Symmetric plane axiom : It can not be distinguished which side of a plane is back or front.

Definition of invariant function : Let $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a matrix, $\boldsymbol{p}=\begin{pmatrix} x \\ y \end{pmatrix}$ be a point, and f be a function of $\boldsymbol{p}$, if $\mathrm{f}(A\boldsymbol{p})=\mathrm{f}(\boldsymbol{p})$, then this function $\mathrm{f}(\boldsymbol{p})$ is called an invariant function of A.

Put oblique coordinate systems on both sides of a plane, and make them coincide with their origins. We define the rear surface coordinate transformation matrix B as follows. Suppose the matrix B transforms from the right-hand to the right-hand system in front of each face side, then $\det B<0$. Any point $\boldsymbol{p}_i=(x_i,y_i)$, i=1,2,3,…on the front side corresponds to the rear point $\boldsymbol{q}_i=(u_i,v_i)$ on the back side as $\boldsymbol{q}_i=B\boldsymbol{p}_i$. If the point $\boldsymbol{p}_i=(x_i,y_i)$ on the back side corresponds to the rear point $\boldsymbol{r}_i=(s_i,t_i)$ on the front side as $\boldsymbol{r}_i=B^{-1}\boldsymbol{p}_i$, and only if $\boldsymbol{q}_i=\boldsymbol{r}_i$ on the front side, then the relationship between any point $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ on the front side has the same relationship between the point $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ on the back side.

Therefore, the symmetric plane equation is

$$\boldsymbol{q}_i=B\boldsymbol{p}_i=B^{-1}\boldsymbol{p}_i \quad \Leftrightarrow \quad B=B^{-1} \quad \Leftrightarrow \quad B^2=E, \quad \text{where } \det B<0. \tag{1}$$

We obtained an oblique reflection matrix B which has two lines belonging to eigen values, and first and second order invariant functions. We shall treat negative solution $-B$ later.

$$B=\pm\begin{pmatrix} a & -b \\ c & -a \end{pmatrix}, \quad \det B=-a^2+bc=-1, \quad \text{eigen values } \lambda=\pm1. \tag{2}$$

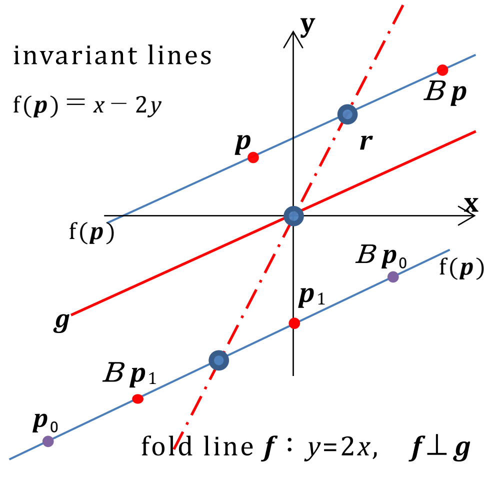
・For eigen value $\lambda=1 \Leftrightarrow B\boldsymbol{p}=\boldsymbol{p}$, we obtained a fold line $f: cx-(a+1)y=0$. (3)

・For eigen value $\lambda=-1 \Leftrightarrow B\boldsymbol{p}=-\boldsymbol{p}$, we obtained an isotropic line g which is parallel to an invariant line $\mathrm{f}(\boldsymbol{p})$ shown as eq(5), $g: cx-(a-1)y=0$. (4)

・We obtained an invariant line $\mathrm{f}(\boldsymbol{p})$: $\mathrm{f}(B\boldsymbol{p})=\mathrm{f}(\boldsymbol{p})=cx-(a-1)y$. (5)

We observed that the middle point $\boldsymbol{r}$ of the point $\boldsymbol{p}$ and $B\boldsymbol{p}$ is in the fold line f. So the invariant line f($\boldsymbol{p}$) is isotropic. Thus, from the back and front symmetric surface, we obtained the oblique reflection matrix B and the oblique reflection planes with both lines of f and g.

Figure Oblique reflection plane



Following figure shows the case of $B = \frac{1}{3}\begin{pmatrix} -5 & 4 \\ -4 & 5 \end{pmatrix}$.

・We obtained the quadratic invariant function ϕ ($\boldsymbol{p}$) of matrix B and A as eq(6). The matrix A is derived from matrix B as follows.

$$A = BM = \begin{pmatrix} a & b \\ c & a \end{pmatrix} = \begin{pmatrix} a & b \\ kb & a \end{pmatrix}, \quad \text{where } M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$\det A = a^2 - kb^2 = 1$, $k = c/b$.

The quadratic invariant function is

$$\phi\,(B\boldsymbol{p}) = \phi\,(A\boldsymbol{p}) = \phi\,(\boldsymbol{p}) = -kx^2 + y^2. \qquad (6)$$

We observed that the Bs and As matrices with any matrix of these combination belongs to the ±special transformation group on the invariant function ϕ ($\boldsymbol{p}$). Also they build the congruent geometry with the metric $\|\boldsymbol{p}\|^2 = \phi\,(\boldsymbol{p}) = -kx^2 + y^2$ on both sides of a symmetric plane. The inverse relation of f and g is the case of the negative solution of the matrix $-B$.

The existing direction of fold lines $\boldsymbol{f}$ belongs to the sign of k such as

when k<0, then fold line $\boldsymbol{f}$: $y = \frac{c}{a+1}x = -\sqrt{-k}\tan\frac{\theta}{2}\cdot x = ux$, $-\infty < u < \infty$, for $\theta \in$ Real.

when k>0, then fold line $\boldsymbol{f}$: $y = \frac{c}{a+1}x = \sqrt{k}\tanh\frac{\theta}{2}\cdot x = ux$, $-\sqrt{k} < u < \sqrt{k}$, for $\theta \in$ Real.

We observed that when the sign of k<0, then this symmetric plane with oblique reflection planes is completely isotropic and fits into space*space plane. And when the sign of k>0, then this symmetric plane with oblique reflection planes is semi isotropic and fits into space*time plane. We remark that when k>0, then this matrix A is called a Lorentz transformation matrix.

Thus, we draw the following conclusions.

・The Pythagorean theorem depends on the symmetry of a space*space plane, such that the squared norm of a vector $\boldsymbol{p}$ is invariant as $\|\boldsymbol{p}\|^2 = \phi\,(\boldsymbol{p}) = -kx^2 + y^2 = x^2 + y^2$, where $k = -1$.

・The special principle of relativity is based on the symmetry of a space*time plane, because this type of a plane is semi isotropic with the sign of k>0. Any basic law of nature with position vectors is subject to the symmety of a plane.

・The arrow of time mystery in physics would be explained since a space*time plane in the cosmos has the symmetry of the back and front surface.

Reference

[1]Hiroaki Fujimori website:Structure of Plane Geometry, http://www.spatim.sakura.ne.jp/